Solve by completing the square.

a. $t^2 - 2t - 3 = 0$

b. $w^2 - 4w - 5 = 0$

c. $h^2 + 6h + 8 = 0$

d. $m^2 + 2m - 15 = 0$

e. $q^2 + 4q - 5 = 0$

f. $p^2 + 2p + 1 = 0$

g. $x^2 + 8x + 7 = 0$

h. $d^2 + 4d - 12 = 0$

i. $s^2 - 8s - 9 = 0$

j. $f^2 - 6f - 16 = 0$

KEYS

a. $t^2 - 2t - 3 = 0$

> With quadratic equations ($ax^2 + bx + c = 0$), you can solve by completing the square.
>
> Step 1: Make sure that the left side of the equation looks like $x^2 + bx$.
>
> Step 2: Add $\left(\frac{b}{2}\right)^2$ to both sides.
>
> Step 3: Factor the left side as $\left(x + \frac{b}{2}\right)^2$
>
> Step 4: Take the square root and solve.

Step 1: Make sure that the left side of the equation looks like $x^2 + bx$.

To make the left side of the equation look like $x^2 + bx$, add 3 to both sides.

$t^2 - 2t - 3 = 0$

$t^2 - 2t = 3$

Step 2: Add $\left(\frac{b}{2}\right)^2$ to both sides.

Since b = -2, $\left(\frac{b}{2}\right)^2 = \left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$. Add 1 to both sides.

$t^2 - 2t + 1 = 4$

Step 3: Factor the left side as $\left(x + \frac{b}{2}\right)^2$.

In general, an expression of the form $x^2 + bx + \left(\frac{b}{2}\right)^2$ can be factored as $\left(x + \frac{b}{2}\right)^2$.

The expression $t^2 - 2t + 1$ is of this form, with b = -2. So, it can be factored as $(t - 1)^2$.

Rewrite the equation with the left side factored.

$(t - 1)^2 = 4$

Step 4: Take the square root and solve.

$t - 1 = \pm 2$ Take the square root.

$t = 1 \pm 2$ Add 1 to both sides.

t = 1 + 2 or t = 1 – 2 Split $\pm$ into + or –

t = 3 or t = – 1 Simplify.

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b. $w^2 – 4w – 5 = 0$

> With quadratic equations ($ax^2 + bx + c = 0$), you can solve by completing the square.
>
> Step 1: Make sure that the left side of the equation looks like $x^2 + bx$.
> Step 2: Add $\left(\frac{b}{2}\right)^2$ to both sides.
> Step 3: Factor the left side as $\left(x+\frac{b}{2}\right)^2$
> Step 4: Take the square root and solve.

Step 1: Make sure that the left side of the equation looks like $x^2 + bx$.

To make the left side of the equation look like $x^2 + bx$, add 5 to both sides.

$w^2 – 4w – 5 = 0$

$w^2 – 4w = 5$

Step 2: Add $\left(\frac{b}{2}\right)^2$ to both sides.

Since b = -4, $\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$. Add 4 to both sides.

$w^2 – 4w + 4 = 9$

Step 3: Factor the left side as $\left(x+\frac{b}{2}\right)^2$.

In general, an expression of the form $x^2 + bx + \left(\frac{b}{2}\right)^2$ can be factored as $\left(x+\frac{b}{2}\right)^2$.

The expression $w^2 – 4w + 4$ is of this form, with b = -4. So, it can be factored as $(w – 2)^2$.

Rewrite the equation with the left side factored.

$(w – 2)^2 = 9$

Step 4: Take the square root and solve.

$w - 2 = \pm 3$ Take the square root.

$w = 2 \pm 3$ Add 2 to both sides.

w = 2 + 3 or w = 2 – 3 Split $\pm$ into + or –

w = 5 or w = – 1 Simplify.

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c. $h^2 + 6h + 8 = 0$

> With quadratic equations ($ax^2 + bx + c = 0$), you can solve by completing the square.
>
> Step 1: Make sure that the left side of the equation looks like $x^2 + bx$.
>
> Step 2: Add $\left(\frac{b}{2}\right)^2$ to both sides.
>
> Step 3: Factor the left side as $\left(x + \frac{b}{2}\right)^2$
>
> Step 4: Take the square root and solve.

Step 1: Make sure that the left side of the equation looks like $x^2 + bx$.

To make the left side of the equation look like $x^2 + bx$, subtract 8 from both sides.

$h^2 + 6h + 8 = 0$

$h^2 + 6h = -8$

Step 2: Add $\left(\frac{b}{2}\right)^2$ to both sides.

Since b = 6, $\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = 3^2 = 9$. Add 9 to both sides.

$h^2 + 6h + 9 = 1$

Step 3: Factor the left side as $\left(x + \frac{b}{2}\right)^2$.

In general, an expression of the form $x^2 + bx + \left(\frac{b}{2}\right)^2$ can be factored as $\left(x + \frac{b}{2}\right)^2$.

The expression $h^2 + 6h + 9$ is of this form, with b = 6. So, it can be factored as $(h + 3)^2$.

Rewrite the equation with the left side factored.

$(h + 3)^2 = 1$

Step 4: Take the square root and solve.

$h + 3 = \pm 1$ Take the square root.

$h = -3 \pm 1$ Subtract 3 from both sides.

h = – 3 + 1 or h = – 3 – 1 Split $\pm$ into + or –

h = – 2 or h = – 4 Simplify.

---

d. $m^2 + 2m - 15 = 0$

> With quadratic equations ($ax^2 + bx + c = 0$), you can solve by completing the square.
>
> Step 1: Make sure that the left side of the equation looks like $x^2 + bx$.
> Step 2: Add $\left(\frac{b}{2}\right)^2$ to both sides.
> Step 3: Factor the left side as $\left(x + \frac{b}{2}\right)^2$
> Step 4: Take the square root and solve.

Step 1: Make sure that the left side of the equation looks like $x^2 + bx$.

To make the left side of the equation look like $x^2 + bx$, add 15 to both sides.

$m^2 + 2m - 15 = 0$

$m^2 + 2m = 15$

Step 2: Add $\left(\frac{b}{2}\right)^2$ to both sides.

Since b = 2, $\left(\frac{b}{2}\right)^2 = \left(\frac{2}{2}\right)^2 = 1^2 = 1$. Add 1 to both sides.

$m^2 + 2m + 1 = 16$

Step 3: Factor the left side as $\left(x + \frac{b}{2}\right)^2$.

In general, an expression of the form $x^2 + bx + \left(\frac{b}{2}\right)^2$ can be factored as $\left(x + \frac{b}{2}\right)^2$.

The expression $m^2 + 2m + 1$ is of this form, with b = 2. So, it can be factored as $(m + 1)^2$. Rewrite the equation with the left side factored.

$(m + 1)^2 = 16$

Step 4: Take the square root and solve.

$m + 1 = \pm 4$ Take the square root.

$m = -1 \pm 4$ Subtract 1 from both sides.

$m = -1 + 4$ or $m = -1 - 4$ Split $\pm$ into + or –

$m = 3$ or $m = -5$ Simplify.

---

e. $q^2 + 4q - 5 = 0$

> With quadratic equations ($ax^2 + bx + c = 0$), you can solve by completing the square.
>
> Step 1: Make sure that the left side of the equation looks like $x^2 + bx$.
>
> Step 2: Add $\left(\frac{b}{2}\right)^2$ to both sides.
>
> Step 3: Factor the left side as $\left(x + \frac{b}{2}\right)^2$
>
> Step 4: Take the square root and solve.

Step 1: Make sure that the left side of the equation looks like $x^2 + bx$.

To make the left side of the equation look like $x^2 + bx$, add 5 to both sides.

$q^2 + 4q - 5 = 0$

$q^2 + 4q = 5$

Step 2: Add $\left(\frac{b}{2}\right)^2$ to both sides.

Since b = 4, $\left(\frac{b}{2}\right)^2 = \left(\frac{4}{2}\right)^2 = 2^2 = 4$. Add 4 to both sides.

$q^2 + 4q + 4 = 9$

Step 3: Factor the left side as $\left(x + \frac{b}{2}\right)^2$.

In general, an expression of the form $x^2 + bx + \left(\frac{b}{2}\right)^2$ can be factored as $\left(x + \frac{b}{2}\right)^2$.

The expression $q^2 + 4q + 4$ is of this form, with b = 4. So, it can be factored as $(q + 2)^2$. Rewrite the equation with the left side factored.

$(q + 2)^2 = 9$

Step 4: Take the square root and solve.

$q + 2 = \pm 3$ Take the square root.

$q = -2 \pm 3$ Subtract 2 from both sides.

q = – 2 + 3 or q = – 2 – 3 Split $\pm$ into + or –

q = 1 or q = – 5 Simplify.

---

f. $p^2 + 2p + 1 = 0$

With quadratic equations ($ax^2 + bx + c = 0$), you can solve by completing the square.

Step 1: Make sure that the left side of the equation looks like $x^2 + bx$.

Step 2: Add $\left(\frac{b}{2}\right)^2$ to both sides.

Step 3: Factor the left side as $\left(x + \frac{b}{2}\right)^2$

Step 4: Take the square root and solve.

Step 1: Make sure that the left side of the equation looks like $x^2 + bx$.

To make the left side of the equation look like $x^2 + bx$, subtract 1 from both sides.

$p^2 + 2p + 1 = 0$

$p^2 + 2p = -1$

Step 2: Add $\left(\frac{b}{2}\right)^2$ to both sides.

Since b = 2, $\left(\frac{b}{2}\right)^2 = \left(\frac{2}{2}\right)^2 = 1^2 = 1$. Add 1 to both sides.

$p^2 + 2p + 1 = 0$

Step 3: Factor the left side as $\left(x + \frac{b}{2}\right)^2$.

In general, an expression of the form $x^2 + bx + \left(\frac{b}{2}\right)^2$ can be factored as $\left(x + \frac{b}{2}\right)^2$.

The expression $p^2 + 2p + 1$ is of this form, with b = 2. So, it can be factored as $(p + 1)^2$. Rewrite the equation with the left side factored.

$(p + 1)^2 = 0$

Step 4: Take the square root and solve.

$p + 1 = \pm 0$ Take the square root.

$p = -1 \pm 0$ Subtract 1 from both sides.

$p = -1 + 0$ or $p = -1 - 0$ Split $\pm$ into + or –

$p = -1$ or $p = -1$ Simplify.

---

g. $x^2 + 8x + 7 = 0$

> With quadratic equations ($ax^2 + bx + c = 0$), you can solve by completing the square.
>
> Step 1: Make sure that the left side of the equation looks like $x^2 + bx$.
> Step 2: Add $\left(\frac{b}{2}\right)^2$ to both sides.
> Step 3: Factor the left side as $\left(x + \frac{b}{2}\right)^2$
> Step 4: Take the square root and solve.

Step 1: Make sure that the left side of the equation looks like $x^2 + bx$.

To make the left side of the equation look like $x^2 + bx$, subtract 7 from both sides.

$x^2 + 8x + 7 = 0$

$x^2 + 8x = -7$

Step 2: Add $\left(\frac{b}{2}\right)^2$ to both sides.

Since b = 8, $\left(\frac{b}{2}\right)^2 = \left(\frac{8}{2}\right)^2 = 4^2 = 16$. Add 16 to both sides.

$x^2 + 8x + 16 = 9$

Step 3: Factor the left side as $\left(x + \frac{b}{2}\right)^2$.

In general, an expression of the form $x^2 + bx + \left(\frac{b}{2}\right)^2$ can be factored as $\left(x + \frac{b}{2}\right)^2$.

The expression $x^2 + 8x + 16$ is of this form, with b = 8. So, it can be factored as $(x + 4)^2$. Rewrite the equation with the left side factored.

$(x + 4)^2 = 9$

Step 4: Take the square root and solve.

$x + 4 = \pm 3$ Take the square root.

$x = -4 \pm 3$ Subtract 4 from both sides.

$x = -4 + 3$ or $x = -4 - 3$ Split $\pm$ into + or –

$x = -1$ or $x = -7$ Simplify.

---

h. $d^2 + 4d - 12 = 0$

> With quadratic equations ($ax^2 + bx + c = 0$), you can solve by completing the square.
>
> Step 1: Make sure that the left side of the equation looks like $x^2 + bx$.
>
> Step 2: Add $\left(\frac{b}{2}\right)^2$ to both sides.
>
> Step 3: Factor the left side as $\left(x + \frac{b}{2}\right)^2$
>
> Step 4: Take the square root and solve.

Step 1: Make sure that the left side of the equation looks like $x^2 + bx$.

To make the left side of the equation look like $x^2 + bx$, add 12 to both sides.

$d^2 + 4d - 12 = 0$

$d^2 + 4d = 12$

Step 2: Add $\left(\frac{b}{2}\right)^2$ to both sides.

Since b = 4, $\left(\frac{b}{2}\right)^2 = \left(\frac{4}{2}\right)^2 = 2^2 = 4$. Add 4 to both sides.

$d^2 + 4d + 4 = 16$

Step 3: Factor the left side as $\left(x + \frac{b}{2}\right)^2$.

In general, an expression of the form $x^2 + bx + \left(\frac{b}{2}\right)^2$ can be factored as $\left(x + \frac{b}{2}\right)^2$.

The expression $d^2 + 4d + 4$ is of this form, with b = 4. So, it can be factored as $(d + 2)^2$. Rewrite the equation with the left side factored.

$(d + 2)^2 = 16$

Step 4: Take the square root and solve.

$d + 2 = \pm 4$ Take the square root.

$d = -2 \pm 4$ Subtract 2 from both sides.

$d = -2 + 4$ or $d = -2 - 4$ Split $\pm$ into + or –

$d = 2$ or $d = -6$ Simplify.

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i. $s^2 - 8s - 9 = 0$

> With quadratic equations ($ax^2 + bx + c = 0$), you can solve by completing the square.
>
> Step 1: Make sure that the left side of the equation looks like $x^2 + bx$.
>
> Step 2: Add $\left(\frac{b}{2}\right)^2$ to both sides.
>
> Step 3: Factor the left side as $\left(x + \frac{b}{2}\right)^2$

Step 4: Take the square root and solve.

Step 1: Make sure that the left side of the equation looks like $x^2 + bx$.

To make the left side of the equation look like $x^2 + bx$, add 9 to both sides.

$s^2 - 8s - 9 = 0$

$s^2 - 8s = 9$

Step 2: Add $\left(\frac{b}{2}\right)^2$ to both sides.

Since b = -8, $\left(\frac{b}{2}\right)^2 = \left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$. Add 16 to both sides.

$s^2 - 8s + 16 = 25$

Step 3: Factor the left side as $\left(x + \frac{b}{2}\right)^2$.

In general, an expression of the form $x^2 + bx + \left(\frac{b}{2}\right)^2$ can be factored as $\left(x + \frac{b}{2}\right)^2$.

The expression $s^2 - 8s + 16$ is of this form, with b = – 8. So, it can be factored as $(s - 4)^2$. Rewrite the equation with the left side factored.

$(s - 4)^2 = 25$

Step 4: Take the square root and solve.

$s - 4 = \pm 5$ Take the square root.

$s = 4 \pm 5$ Add 4 to both sides.

s = 4 + 5 or s = 4 – 5 Split $\pm$ into + or –

s = 9 or s = – 1 Simplify.

---

j. $f^2 - 6f - 16 = 0$

With quadratic equations ($ax^2 + bx + c = 0$), you can solve by completing the square.

Step 1: Make sure that the left side of the equation looks like $x^2 + bx$.
Step 2: Add $\left(\frac{b}{2}\right)^2$ to both sides.
Step 3: Factor the left side as $\left(x + \frac{b}{2}\right)^2$
Step 4: Take the square root and solve.

Step 1: Make sure that the left side of the equation looks like $x^2 + bx$.

To make the left side of the equation look like $x^2 + bx$, add 16 to both sides.

$f^2 - 6f - 16 = 0$

$f^2 - 6f = 16$

Step 2: Add $\left(\frac{b}{2}\right)^2$ to both sides.

Since b = – 6, $\left(\frac{b}{2}\right)^2 = \left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$. Add 9 to both sides.

$f^2 - 6f + 9 = 25$

Step 3: Factor the left side as $\left(x + \frac{b}{2}\right)^2$.

In general, an expression of the form $x^2 + bx + \left(\frac{b}{2}\right)^2$ can be factored as $\left(x + \frac{b}{2}\right)^2$.

The expression $f^2 - 6f + 9$ is of this form, with b = – 6. So, it can be factored as $(f - 3)^2$. Rewrite the equation with the left side factored.

$(f - 3)^2 = 25$

Step 4: Take the square root and solve.

$f - 3 = \pm 5$ Take the square root.

$f = 3 \pm 5$ Add 3 to both sides.

f = 3 + 5 or s = 3 – 5 Split $\pm$ into + or –

f = 8 or s = – 2 Simplify.